Lambek Grammars Are Context Free

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Abstract

In this paper the Chomsky Conjecture is proved: all languages recognized by the Lambek calculus are context free.

Introduction

The notion of a basic categorial grammar was introduced in [1]. In the same paper it was proved that basic categorial grammars are precisely the context-free ones.

Another kind of categorial grammars was introduced by J. Lambek [8]. These grammars are based on a syntactic calculus, presently known as the Lambek calculus (cf. [2] for its semantic interpretations). Chomsky [6] conjectured that these grammars are also equivalent to context-free ones. In [7] Cohen proved that every basic categorial grammar (and, thus, every context-free grammar) is equivalent to a Lambek grammar. He also proposed a proof of the converse. However, as pointed out in [3], this proof contains an error. Buszkowski proved that some special kinds of Lambek grammars are context-free [3, 4, 5]. These grammars use weakly unidirectional types or types of order at most two.

The main result of this paper (Theorem 2) says that Lambek grammars generate only context-free languages. Thus they are equivalent to context-free grammars and also to basic categorial grammars.

1 Preliminaries

1.1 Lambek calculus

We consider the syntactic calculus introduced in [8]. The types of the Lambek calculus are built of primitive types $p_1, p_2, \ldots$, and three binary connectives $\bullet, \backslash, /$. We shall denote the set of all types by Tp. Capital letters A,B,... range over types. Capital Greek letters range over finite (possibly empty) sequences of types. Sequents of the Lambek calculus are of the form $\Gamma\to A$, where Γ is a nonempty sequence of types.

Axioms: $p_i \to p_i$

Rules:

$$\frac{\Gamma\to A \qquad \Delta\to B}{\Gamma\ \Delta\to A\bullet B}\ (\to\bullet)$$

$$\frac{\Gamma\ A\ B\ \Delta\to C}{\Gamma\ A\bullet B\ \Delta\to C}\ (\bullet\to)$$

$$\frac{A\ \Pi\to B}{\Pi\to A\backslash B}\ (\to\backslash) \quad \text{where } \Pi \text{ is not empty}$$

$$\frac{\Pi\ A\to B}{\Pi\to B/A}\ (\to/) \quad \text{where } \Pi \text{ is not empty}$$

$$\frac{\Phi\to A \qquad \Gamma\ B\ \Delta\to C}{\Gamma\ \Phi\ A\backslash B\ \Delta\to C}\ (\backslash\to)$$

$$\frac{\Phi\to A \qquad \Gamma\ B\ \Delta\to C}{\Gamma\ B/A\ \Phi\ \Delta\to C}\ (/\to)$$

$$\frac{\Phi\to B \qquad \Gamma\ B\ \Delta\to A}{\Gamma\ \Phi\ \Delta\to A}\ (\text{CUT})$$

The cut-elimination theorem for this calculus is proved in [8].

We write $L\vdash\Gamma\to A$ if the sequent $\Gamma\to A$ is derivable in the Lambek calculus.

Definition. The *length* of a type is defined as the total number of primitive type occurrences in the type.

$$||p_i|| \rightleftharpoons 1 \quad ||A\bullet B|| = ||A\backslash B|| = ||A/B|| \rightleftharpoons ||A|| + ||B||$$

1.2 Lambek grammars and context-free grammars

Definition. We assume that a finite alphabet $\mathcal{T}$ and a distinguished type D are given. A *Lambek grammar* is a mapping f such that, for all $t \in \mathcal{T}$, $f(t) \subset Tp$ and $f(t)$ is finite.

The *language generated by the Lambek grammar* is defined as the set of all expressions $t_1 \dots t_n$ over the alphabet $\mathcal{T}$ for which there exists a derivable sequent $B_1 \ \dots \ B_n \to D$ such that $B_i \in f(t_i)$ for all $i \le n$. We shall denote this language by $\mathcal{L}(\mathcal{T}, D, f)$.

Definition. We assume that two disjoint alphabets $\mathcal{T}$ and $\mathcal{W}$ are given. The elements of $\mathcal{T}$ are called *terminal symbols* and those of $\mathcal{W}$ are *auxiliary symbols*.

A *context-free rewrite rule* is of the form $X \Rightarrow e$, where X is an auxiliary symbol and e is a word in the alphabet $\mathcal{T} \cup \mathcal{W}$.

A *context-free grammar* is a finite set $\mathcal{R}$ of context-free rewrite rules, with one auxiliary symbol S designated as its *start symbol*.

By $\bar{\mathcal{G}}(\mathcal{T}, \mathcal{W}, S, \mathcal{R})$ we denote the set of all expressions over the alphabet $\mathcal{T} \cup \mathcal{W}$ that arise through some finite sequence of rewritings of the start symbol S via the rules of $\mathcal{R}$.

The *language generated by the context-free grammar* is defined as

$$\mathcal{G}(\mathcal{T}, \mathcal{W}, S, \mathcal{R}) \rightleftharpoons \bar{\mathcal{G}}(\mathcal{T}, \mathcal{W}, S, \mathcal{R}) \cap \mathcal{T}^+,$$

where $\mathcal{T}^+$ denotes the set of all nonempty expressions over the alphabet $\mathcal{T}$.

2 Main result

In this section we show that every language recognized by a Lambek grammar can also be generated by a context-free grammar. The crucial point is that every sequent $B_1 \ \dots \ B_n \to D$ derivable in the Lambek calculus follows immediately (i.e., by means of the cut rule only) from some short derivable sequents containing at most three types each, where none of the types is longer than the longest type in $B_1 \ \dots \ B_n \to D$ (cf. Theorem 1). The proof of Theorem 1 will be carried out later in Section 3.

In order to formalize the notion of immediate consequence, we introduce for each natural number m two calculi $Lcut_m$ and $Lcut_m^-$.

Definition. A sequent $A_1 \ \dots \ A_n \to B$ is an axiom of $Lcut_m$ iff

(1) $n \le 2$;

(2) the sequent $A_1 \ \dots \ A_n \to B$ is derivable in the Lambek calculus;

(3) $||B|| \le m$ and $||A_i|| \le m$ for all $i \le n$.

The only rule of $Lcut_m$ is (CUT).

Definition. The calculus $Lcut_m^-$ has the same axioms as $Lcut_m$. The only rule of $Lcut_m^-$ is (CUT) with the restriction that the left premise $\Phi \to B$ must be an axiom of $Lcut_m^-$.

Theorem 1 *$Lcut_m \vdash B_1 \ \dots \ B_n \to D$ if and only if $||B_i|| \le m$ for all $i \le n$, $||D|| \le m$, and $L \vdash B_1 \ \dots \ B_n \to D$.*

The theorem will be proved in Section 3.

Lemma 1 *$Lcut_m^- \vdash \Phi \to A$ if and only if $Lcut_m \vdash \Phi \to A$.*

PROOF. The 'only if' part is obvious. The 'if' part is proved by induction on the length of the $Lcut_m^-$-derivation of the left premise of a cut. A derivation of the form

$$\dfrac{\dfrac{\Phi \to B \qquad \Gamma\ B\ \Delta \to A}{\Gamma\ \Phi\ \Delta \to A}\,(\mathrm{CUT}) \qquad \Theta\ A\ \Pi \to C}{\Theta\ \Gamma\ \Phi\ \Delta\ \Pi \to C}\,(\mathrm{CUT})$$

will be rearranged in the following way.

$$\dfrac{\Phi \to B \qquad \dfrac{\Gamma\ B\ \Delta \to A \qquad \Theta\ A\ \Pi \to C}{\Theta\ \Gamma\ B\ \Delta\ \Pi \to C}\,(\mathrm{CUT})}{\Theta\ \Gamma\ \Phi\ \Delta\ \Pi \to C}\,(\mathrm{CUT})$$

■

Theorem 2 *For any Lambek grammar there exists a context-free grammar such that the languages generated by these grammars coincide.*

PROOF. If a fixed alphabet $\mathcal{T}$, a designated type D, and a mapping f are given, then the set of types relevant in the definition of $\mathcal{L}(\mathcal{T}, D, f)$ is finite. Let m be the maximum of the lengths of these types. Then $||D|| \le m$ and, for any $t \in \mathcal{T}$, for any $B \in f(t)$, $||B|| \le m$.

The set of primitive types involved in the grammar is also finite. Below we shall consider only types consisting of these primitive types.

We take as the alphabet of auxiliary symbols $\mathcal{W}$ the set of all types not longer than m (and containing only relevant primitive types).

$$\mathcal{W} \rightleftharpoons \{A \in Tp \mid ||A|| \le m\}$$

We take the distinguished type D as the start symbol of the context-free grammar.

The set $\mathcal{R}$ consists of obvious rules describing the mapping f and of $Lcut_m$-axioms with the sequent arrows reversed.

$$\mathcal{R} \rightleftharpoons \{B \Rightarrow t \mid t \in \mathcal{T} \text{ and } B \in f(t)\} \cup$$
$$\cup \{A \Rightarrow BC \mid A, B, C \in \mathcal{W} \text{ and } L \vdash B\ C \rightarrow A\} \cup$$
$$\cup \{A \Rightarrow B \mid A, B \in \mathcal{W} \text{ and } L \vdash B \rightarrow A\}$$

First, we prove that $\mathcal{L}(\mathcal{T}, D, f) \subset \mathcal{G}(\mathcal{T}, \mathcal{W}, D, \mathcal{R})$. Suppose that $t_1 \dots t_n \in \mathcal{L}(\mathcal{T}, D, f)$. According to the definition of $\mathcal{L}(\mathcal{T}, D, f)$ there are types $B_1, \dots, B_n$ such that $L \vdash B_1\ \dots\ B_n \rightarrow D$ and $B_i \in f(t_i)$ for all $i \leq n$. By construction, $B_i \Rightarrow t_i \in \mathcal{R}$ for all $i \leq n$. Thus it suffices to prove that $B_1 \dots B_n \in \bar{\mathcal{G}}(\mathcal{T}, \mathcal{W}, D, \mathcal{R})$.

In view of Theorem 1 and Lemma 1, $Lcut_m^- \vdash B_1\ \dots\ B_n \rightarrow D$.
Straightforward induction on the length of a $Lcut_m^-$-derivation shows that if $Lcut_m^- \vdash B_1\ \dots\ B_n \rightarrow D$ then $B_1 \dots B_n \in \bar{\mathcal{G}}(\mathcal{T}, \mathcal{W}, D, \mathcal{R})$.

Now we prove that $\mathcal{G}(\mathcal{T}, \mathcal{W}, D, \mathcal{R}) \subset \mathcal{L}(\mathcal{T}, D, f)$. We extend the mapping f to the set $\mathcal{T} \cup \mathcal{W}$ stipulating $f(B) = \{B\}$ for all $B \in \mathcal{W}$. Easy induction on the number of rewritings establishes that if an expression $X_1 \dots X_n$ over the alphabet $\mathcal{T} \cup \mathcal{W}$ belongs to $\bar{\mathcal{G}}(\mathcal{T}, \mathcal{W}, D, \mathcal{R})$ then there are auxiliary symbols $B_1, \dots, B_n$ such that $B_i \in f(X_i)$ for all $i \leq n$, and $Lcut_m^- \vdash B_1\ \dots\ B_n \rightarrow D$. In particular, if $t_1, \dots, t_n$ are terminal symbols and $t_1 \dots t_n$ belongs to $\bar{\mathcal{G}}(\mathcal{T}, \mathcal{W}, D, \mathcal{R})$ then $t_1 \dots t_n \in \mathcal{L}(\mathcal{T}, D, f)$. ■

3 Proof of Theorem 1

Let FG stand for the free group generated by all primitive types $\{p_i \mid i \in \mathbf{N}\}$. The identity element will be denoted by ϵ. For any element $u \in FG$, we write $|u|$ for the length of u written as a reduced word, i.e., a word that does not contain any fragments of the form $p_i p_i^{-1}$ or $p_i^{-1} p_i$.

Definition. The *free group interpretation* of types (written as $[\![\ ]\!]$) is the following mapping of types into FG.

$$\begin{aligned} [\![p_i]\!] &\rightleftharpoons p_i \\ [\![A \bullet B]\!] &\rightleftharpoons [\![A]\!] \circ [\![B]\!] \\ [\![A \backslash B]\!] &\rightleftharpoons [\![A]\!]^{-1} \circ [\![B]\!] \\ [\![A / B]\!] &\rightleftharpoons [\![A]\!] \circ [\![B]\!]^{-1} \\ [\![A_1 \dots A_n]\!] &\rightleftharpoons [\![A_1]\!] \circ \dots \circ [\![A_n]\!] \end{aligned}$$

Lemma 2 *If a sequent $\Gamma \rightarrow C$ is derivable in the Lambek calculus, then $[\![\Gamma]\!] = [\![C]\!]$.*

D. Roorda obtained this result in terms of atomic markings. The lemma has also an immediate proof in the free group environment [9].

Definition. For each natural number i we define the counter of occurrences of the primitive type p_i as follows.

$$\sigma_i p_i \rightleftharpoons 1 \qquad \sigma_i p_j \rightleftharpoons 0, \text{ if } i \neq j$$

$$\sigma_i(A \bullet B) = \sigma_i(A \backslash B) = \sigma_i(A/B) \rightleftharpoons \sigma_i A + \sigma_i B$$

We extend this definition to finite sequences of types.

$$\sigma_i(A_1 \dots A_n) \rightleftharpoons \sigma_i A_1 + \dots + \sigma_i A_n$$

Remark. If a sequent $\Gamma \rightarrow A$ is derivable in the Lambek calculus, then for any i, $\sigma_i(\Gamma A)$ is an even number.

Remark. $||A|| = \sum_i \sigma_i A$

Definition. A sequent $\Gamma \rightarrow A$ is *thin* iff

(1) $\Gamma \rightarrow A$ is derivable in the Lambek calculus;

(2) $\sigma_i(\Gamma A) = 2$ for all primitive types p_i occurring in $\Gamma \rightarrow A$.

Lemma 3 *If $L \vdash \Gamma\ \Phi\ \Delta \rightarrow C$ then there is a type B (an interpolant for Φ in $\Gamma\ \Phi\ \Delta \rightarrow C$) such that*

(i) $L \vdash \Phi \rightarrow B$;

(ii) $L \vdash \Gamma\ B\ \Delta \rightarrow C$.

(iii) $\sigma_i B \leq min(\sigma_i \Phi, \sigma_i(\Gamma \Delta C))$;

Proof. This lemma is a slight modification of the Interpolation theorem for the Lambek calculus proved by Dirk Roorda in [10, p. 84]. The only difference is that Roorda allows also sequents with empty antecedents to occur in derivations. We omit the straightforward proof by induction on a cut-free derivation of $\Gamma\ \Phi\ \Delta \rightarrow C$. ■

Lemma 4 *If a sequent $\Gamma\ \Phi\ \Delta \rightarrow C$ is thin, then there is a type B such that*

(i) *the sequent $\Phi \rightarrow B$ is thin;*

(ii) *the sequent $\Gamma\ B\ \Delta \rightarrow C$ is thin;*

(iii) $||B|| = |[\![\Phi]\!]|$, *i.e., the length of B equals to the length of the reduced word for the free group interpretation of Φ.*

Proof.
For all i, $\sigma_i\Phi + \sigma_i(\Gamma\Delta C)$ equals either 0 or 2.
If $\sigma_i\Phi = 0$ then $\min(\sigma_i\Phi, \sigma_i(\Gamma\Delta C)) = 0$.
If $\sigma_i\Phi = 1$ then $\min(\sigma_i\Phi, \sigma_i(\Gamma\Delta C)) = 1$.
If $\sigma_i\Phi = 2$ then $\min(\sigma_i\Phi, \sigma_i(\Gamma\Delta C)) = 0$.
According to Lemma 3 there is an interpolant B containing at most one occurrence of every literal.

For any i,

$$\sigma_i(\Phi B) = \sigma_i\Phi + \sigma_i B \leq \sigma_i(\Gamma\Phi\Delta C) + \sigma_i B \leq 2 + 1.$$

Since $\sigma_i(\Phi B)$ is even, we conclude that $\sigma_i(\Phi B)$ is either 0 or 2. This proves (i). The claim (ii) is proved similarly.

According to Lemma 2 $[\![\Gamma]\!][\![\Phi]\!][\![\Delta]\!] = [\![C]\!]$, whence $[\![\Phi]\!] = [\![\Gamma]\!]^{-1}[\![C]\!][\![\Delta]\!]^{-1}$. We conclude that the reduced words for $[\![\Phi]\!]$ and $[\![\Gamma]\!]^{-1}[\![C]\!][\![\Delta]\!]^{-1}$ coincide.

If $\sigma_i\Phi = 0$ then the letter p_i does not occur in $[\![\Phi]\!]$. If $\sigma_i\Phi = 1$ then there is exactly one occurrence of p_i in $[\![\Phi]\!]$. If $\sigma_i\Phi = 2$, then $\sigma_i(\Gamma\Delta C) = 0$, whence p_i does not occur in $[\![\Gamma]\!]^{-1}[\![C]\!][\![\Delta]\!]^{-1}$ and consequently it has no occurrences in the reduced word for $[\![\Phi]\!]$.

We have verified that the reduced word for $[\![\Phi]\!]$ contains exactly the literals that occur in the type B. We also see that no literal has more than one occurrence in the reduced word. This proves statement (iii). ■

Lemma 5 *If no type in a thin sequent* $A_1\ \ldots\ A_n \to C$ *is longer than* m*, then* $Lcut_m \vdash A_1\ \ldots\ A_n \to C$.

To prove Lemma 5, we need some facts about lengths of reduced words in a free group.

Lemma 6 *If* $u, v, w \in FG$ *and* $|uv| > |v|$*, then* $|uvw| > |vw|$.

Proof. Given two reduced words u and v, there exist reduced words a, b, c such that $u = ab^{-1}$, $v = bc$, $uv = ac$, and the words ab^{-1}, bc, ac are reduced. Similarly for v and w there exist reduced words d, e ,and f such that $v = de$, $w = e^{-1}f$, $vw = df$, where de, $e^{-1}f$, df are reduced.

We consider two cases.

Case 1: $|b| \leq |d|$
Evidently $d = bg$, where g is a reduced word.

$$uvw = \underbrace{ab^{-1}}_{u}\,\underbrace{bge}_{v}\,\underbrace{e^{-1}f}_{w}$$

Obviously
$|uv| = |a| + |g| + |e|$, $|vw| = |b| + |g| + |f|$, and $|uvw| = |a| + |g| + |f|$. The assumption of the lemma implies

$$|a| + |g| + |e| > |b| + |g| + |e|,$$

whence $|a| > |b|$. Thus

$$|uvw| = |a| + |g| + |f| > |b| + |g| + |f| = |vw|.$$

Case 2: $|b| > |d|$
Evidently $b = dh$, where h is a reduced word.

$$uvw = \underbrace{ah^{-1}d^{-1}}_{u}\,\underbrace{dhc}_{v}\,\underbrace{c^{-1}h^{-1}f}_{w}$$

Obviously $uvw = ah^{-1}f$ and $ah^{-1}f$ is a reduced word. The assumption of the lemma entails

$$|a| + |c| > |d| + |h| + |c|,$$

whence $|a| > |d| + |h|$. Thus

$$|uvw| = |a| + |h| + |f| > |d| + |h| + |h| + |f| >$$

$$> |d| + |f| = |vw|.$$

■

Lemma 7 *If* $u, v, w \in FG$*,* $|uv| > \max(|u|, |v|)$*, and* $|vw| > \max(|v|, |w|)$*, then* $|uvw| > \max(|u|, |vw|)$.

Proof. We verify that $|uvw| > |vw|$ and $|uvw| > |u|$.

First, $|uv| > |v|$ implies $|uvw| > |vw|$ according to Lemma 6. Dually, $|vw| > |w|$ implies $|uvw| > |uv|$. Thus, in view of $|uv| > |u|$, we conclude that $|uvw| > |u|$. ■

Lemma 8 *If* $u_1, \ldots, u_n \in FG$*,* $n > 1$*, and* $u_1 \ldots u_n = \epsilon$*, then there is a number* $k < n$ *such that* $|u_k u_{k+1}| \leq \max(|u_k|, |u_{k+1}|)$.

Proof. Induction on n. The case $n = 2$ is obvious, since $|u_1u_2| = |\epsilon| = 0$.

Now we prove the lemma for $n+1$ words $u_1, \ldots, u_{n-1}, v, w$, assuming that it holds for any sequence of n words.

Suppose that $|u_{n-1}v| > \max(|u_{n-1}|, |v|)$ and $|vw| > \max(|v|, |w|)$. By Lemma 7,

$$|u_{n-1}vw| > \max(|u_{n-1}|, |vw|). \tag{1}$$

Applying the induction hypothesis for $u_1, \ldots, u_{n-1}, u_n$, where $u_n = vw$, we find a number $k < n$ such that $|u_k u_{k+1}| \leq \max(|u_k|, |u_{k+1}|)$. In view of (1), $k \neq n-1$. This completes the proof. ■

Proof of Lemma 5. Induction on n. If $n \leq 2$ then $A_1 \ \ldots \ A_n \to C$ is an axiom of $Lcut_m$.

Assume that $n > 2$. In view of Lemma 2, $[\![A_1]\!] \ldots [\![A_n]\!][\![C]\!]^{-1} = \epsilon$. We apply Lemma 8 for $u_1 = [\![A_1]\!], \ldots, u_n = [\![A_n]\!], u_{n+1} = [\![C]\!]^{-1}$. Evidently $|u_i| \leq m$ for all $i \leq n+1$. According to Lemma 8 there is a number $k \leq n$ such that $|u_k u_{k+1}| \leq m$. The following two cases arise.

Case 1: $k < n$
This means that $|[\![A_k A_{k+1}]\!]| \leq m$. Applying Lemma 4 for

$$\underbrace{A_1 \ldots A_{k-1}}_{\Gamma} \underbrace{A_k A_{k+1}}_{\Phi} \underbrace{A_{k+2} \ldots A_n}_{\Delta} \to C$$

we find a type B such that $A_k \ A_{k+1} \to B$ is thin, $A_1 \ \ldots \ A_{k-1} \ B \ A_{k+2} \ \ldots \ A_n \to C$ is thin, and $||B|| = |[\![A_k A_{k+1}]\!]| = |u_k u_{k+1}| \leq m$.

Note that $A_{k-1} \ A_k \to B$ is an axiom of $Lcut_m$. Now we use the induction hypothesis for the sequent $A_1 \ \ldots \ A_{k-1} \ B \ A_{k+2} \ \ldots \ A_n \to C$ and after that apply the cut rule.

Case 2: $k = n$
This means that $|[\![A_n]\!][\![C]\!]^{-1}| \leq m$. Applying Lemma 4 for

$$\underbrace{A_1 \ldots A_{n-1}}_{\Phi} \underbrace{A_n}_{\Delta} \to C$$

we find a type B such that $A_1 \ \ldots \ A_{n-1} \to B$ is thin, $B \ A_n \to C$ is thin, and $||B|| = |[\![A_1 \ldots A_{n-1}]\!]|$.
In view of Lemma 2,

$$[\![A_1 \ldots A_{n-1}]\!] = ([\![A_n]\!][\![C]\!]^{-1})^{-1} = (u_n u_{n+1})^{-1}.$$

Thus $||B|| = |(u_n u_{n+1})^{-1}| = |u_n u_{n+1}| \leq m$.

Hence $B \ A_n \to C$ is an axiom of $Lcut_m$. By the induction hypothesis, $Lcut_m \vdash A_1 \ \ldots \ A_{n-1} \to B$. This completes the proof of Lemma 5. ■

We prove now that $Lcut_m \vdash B_1 \ \ldots \ B_n \to D$ if and only if $||B_i|| \leq m$ for all $i \leq n$, $||D|| \leq m$, and $L \vdash B_1 \ \ldots \ B_n \to D$.

Proof of Theorem 1. The 'only if' part is obvious. To prove the 'if' part, we assume that $||B_i|| \leq m$ for all $i \leq n$, $||D|| \leq m$, and $L \vdash B_1 \ \ldots \ B_n \to D$.

We introduce a new primitive type for each instance of an axiom in the derivation of $B_1 \ \ldots \ B_n \to D$ and replace all occurrences of old primitive types in the derivation by corresponding new ones. We obtain a derivation of a sequent $\hat{B}_1 \ \ldots \ \hat{B}_n \to \hat{D}$, where $\sigma_i(\hat{B}_1 \ldots \hat{B}_n \hat{D}) = 2$ for all primitive types p_i occurring in $\hat{B}_1 \ \ldots \ \hat{B}_n \to \hat{D}$.

In view of Lemma 5, $Lcut_m \vdash \hat{B}_1 \ \ldots \ \hat{B}_n \to \hat{D}$. Replacing new primitive types by corresponding old ones in this $Lcut_m$-derivation, we obtain a $Lcut_m$-derivation of $B_1 \ \ldots \ B_n \to D$. This completes the proof of Theorem 1. ■

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References

[1] Y. Bar-Hillel, C. Gaifman, and E. Shamir. On categorial and phrase-structure grammars. *Bull. Res. Council Israel Sect. F*, 9F:1–16, 1960.

[2] J. van Benthem. *Language in Action*. North-Holland, Amsterdam, 1991.

[3] W. Buszkowski. The equivalence of unidirectional Lambek categorial grammars and context-free grammars. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 31:369–384, 1985.

[4] W. Buszkowski. Generative power of categorial grammars. In R.T. Oehrle, E. Bach, and D. Wheeler, editors, *Categorial Grammars and Natural Language Structures*, pages 69–94, Reidel, Dordrecht, 1988.

[5] W. Buszkowski. On generative capacity of the Lambek calculus. In J. van Eijck, editor, *Logics in AI*, pages 139–152, Springer, Berlin, 1991.

[6] N. Chomsky. Formal properties of grammars. In R.D. Luce et al., editors, *Handbook of Mathematical Psychology*, vol. 2, pages 323–418, Wiley, New York, 1963.

[7] J.M. Cohen. The equivalence of two concepts of categorial grammar. *Information and Control*, 10:475–484, 1967.

[8] J. Lambek. The mathematics of sentence structure. *American Mathematical Monthly*, 65(3):154–170, 1958.

[9] M. Pentus. *Equivalent Types in Lambek Calculus and Linear Logic.* Preprint No.2 of the Department of Math. Logic, Steklov Math. Institute, Series Logic and Computer Science, Moscow, 1992.

[10] D. Roorda. *Resource Logics: Proof-theoretical Investigations.* PhD thesis, Fac. Math. and Comp. Sc., University of Amsterdam, 1991.